

Image Enhancement through Discrete and Stationary Wavelet Transforms
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Abstracts

In this correspondence, an image resolution enhancement technique based on interpolation of the high frequency sub-band images obtained by discrete wavelet transform (DWT) and the input image. The edges are enhanced by introducing an intermediate stage by using stationary wavelet transform (SWT). DWT is applied in order to decompose an input image into different sub-bands. Then the high frequency sub-bands as well as the input image are interpolated. The estimated high frequency sub-bands are being modified by using high frequency sub-band obtained through SWT. Then all these sub-bands are combined to generate a new high resolution image by using inverse DWT (IDWT). The quantitative and visual results are showing the superiority of the proposed technique over the conventional and state-of-art image resolution enhancement techniques.

Keywords: DWT, IDWT, SWT, IMAGE, EDGE.

Introduction

Resolution has been frequently referred as an important aspect of an image. Images are being processed in order to obtain more enhanced resolution. One of the commonly used techniques for image resolution enhancement is Interpolation. Interpolation has been widely used in many image processing applications such as facial reconstruction, multiple description coding, and super resolution. There are three well known interpolation techniques, namely nearest neighbor interpolation, bilinear interpolation, and bi-cubic interpolation.

Image resolution enhancement in the wavelet domain is a relatively new research topic and recently many new algorithms have been proposed. Discrete wavelet transform (DWT) is one of the recent wavelet transforms used in image processing. DWT decomposes an image into different sub band images, namely low-low (LL), low high (LH), high-low (HL), and high-high (HH). Another recent wavelet transform which has been used in several image processing applications is stationary wavelet transform (SWT). In short, SWT is similar to DWT but it does not use down-sampling, hence the sub bands will have the same size as the input image.

In this work, we are proposing an image resolution enhancement technique which generates sharper high resolution image. The proposed technique uses DWT to decompose a low resolution image into different sub bands. Then the three high frequency sub band images

have been interpolated using bicubic interpolation. The high frequency sub bands obtained by SWT of the input image are being incremented into the interpolated high frequency sub bands in order to correct the estimated coefficients. In parallel, the input image is also interpolated separately.

Finally, corrected interpolated high frequency sub bands and interpolated input image are combined by using inverse DWT (IDWT) to achieve a high resolution output image. The proposed technique has been compared with conventional and state-of-art image resolution enhancement techniques. The conventional techniques used are the following: interpolation techniques: bilinear interpolation and bi-cubic interpolation, Wavelet zero padding (WZP).

Digital image processing is an area characterized by the need for extensive experimental work to establish the viability of proposed solutions to a given problem. An important characteristic underlying the design of image processing systems is the significant level of testing & experimentation that normally is required before arriving at an acceptable solution. This characteristic implies that the ability to formulate approaches & quickly prototype candidate solutions generally plays a major role in reducing the cost & time required to arrive at a viable system implementation.

An image is represented as a two dimensional function $f(x, y)$ where x and y are spatial co-ordinates and the amplitude of 'f' at any pair of coordinates (x, y) is called the intensity of the image at that point

Wavelets

The Wavelet transform is a transform of this type. It provides the time-frequency representation. (There are other transforms which give this information too, such as short time Fourier transforms, Wigner distributions, etc.) Often times a particular spectral component occurring at any instant can be of particular interest. In these cases it may be very beneficial to know the time intervals these particular spectral components occur. For example, in EEGs, the latency of an event-related potential is of particular interest (Event-related potential is the response of the brain to a specific stimulus like flash-light, the latency of this response is the amount of time elapsed between the onset of the stimulus and the response).

Wavelet transform is capable of providing the time and frequency information simultaneously, hence giving a time-frequency representation of the signal. How wavelet transform works is completely a different fun story, and should be explained after short time Fourier Transform (STFT). The WT was developed as an alternative to the STFT. The STFT will be explained in great detail in the second part of this tutorial. It suffices at this time to say that the WT was developed to overcome some resolution related problems of the STFT, as explained in Part II. To make a real long story short, we pass the time-domain signal from various high pass and low pass filters, which filter out either high frequency or low frequency portions of the signal. This procedure is repeated, every time some portion of the signal corresponding to some frequencies being removed from the signal.

Here is how this works: Suppose we have a signal which has frequencies up to 1000 Hz. In the first stage we split up the signal in to two parts by passing the signal from a high pass and a low pass filter (filters should satisfy some certain conditions, so-called admissibility condition) which results in two different versions of the same signal: portion of the signal corresponding to 0-500 Hz (low pass portion), and 500-1000 Hz (high pass portion). Then, we take either portion (usually low pass portion) or both, and do the same thing again. This operation is called decomposition.

Assuming that we have taken the low pass portion, we now have 3 sets of data, each corresponding to the

same signal at frequencies 0-250 Hz, 250-500 Hz, 500-1000 Hz. Then we take the low pass portion again and pass it through low and high pass filters; we now have 4 sets of signals corresponding to 0-125 Hz, 125-250 Hz, 250-500 Hz, and 500-1000 Hz. We continue like this until we have decomposed the signal to a pre-defined certain level. Then we have a bunch of signals, which actually represent the same signal, but all corresponding to different frequency bands. We know which signal corresponds to which frequency band, and if we put all of them together and plot them on a 3-D graph, we will have time in one axis, frequency in the second and amplitude in the third axis. This will show us which frequencies exist at which time (there is an issue, called "uncertainty principle", which states that, we cannot exactly know what frequency exists at what time instance, but we can only know what frequency bands exist at what time intervals).

The uncertainty principle, originally found and formulated by Heisenberg, states that, the momentum and the position of a moving particle cannot be known simultaneously. This applies to our subject as follows: The frequency and time information of a signal at some certain point in the time-frequency plane cannot be known.

In other words: We cannot know what spectral component exists at any given time instant. The best we can do is to investigate what spectral components exist at any given interval of time. This is a problem of resolution, and it is the main reason why researchers have switched to WT from STFT. STFT gives a fixed resolution at all times, whereas WT gives a variable resolution as follows: Higher frequencies are better resolved in time, and lower frequencies are better resolved in frequency. This means that, a certain high frequency component can be located better in time (with less relative error) than a low frequency component. On the contrary, a low frequency component can be located better in frequency compared to high frequency component.

The fourier transform

In 19th century (1822*, to be exact, but you do not need to know the exact time. Just trust me that it is far before than you can remember), the French mathematician J. Fourier, showed that any periodic function can be expressed as an infinite sum of periodic complex exponential functions. Many years after he had discovered this remarkable property of (periodic) functions, his ideas were generalized to first non-periodic functions, and then periodic or non-periodic discrete time signals. It is after this generalization that it became a very suitable tool for

computer calculations. In 1965, a new algorithm called fast Fourier Transform (FFT) was developed and FT became even more popular. Now let us take a look at how Fourier transform works: FT decomposes a signal to complex exponential functions of different frequencies. The way it does this, is defined by the following two equations:

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-2\pi jft} dt \dots \dots (1)$$

$$x(t) = \int_{-\infty}^{\infty} X(f) \cdot e^{2\pi jft} df \dots \dots (2)$$

In the above equation, t stands for time, f stands for frequency, and x denotes the signal at hand. Note that x denotes the signal in time domain and the X denotes the signal in frequency domain. This convention is used to distinguish the two representations of the signal. FT Equation (1) is called the Fourier transform of x(t), and FT equation (2) is called the inverse Fourier transform of X(f), which is x(t). For those of you who have been using the Fourier transform are already familiar with this. Unfortunately many people use these equations without knowing the underlying principle.

The signal x(t), is multiplied with an exponential term, at some certain frequency "f", and then integrated over ALL TIMES !!! Note that the exponential term in FT Eqn. (1) can also be written as: $\cos(2\pi f t) + j \sin(2\pi f t) \dots \dots (3)$. The above expression has a real part of cosine of frequency f, and an imaginary part of sine of frequency f. So what we are actually doing is, multiplying the original signal with a complex expression which has sines and cosines of frequency f. Then we integrate this product. In other words, we add all the points in this product. If the result of this integration (which is nothing but some sort of infinite summation) is a large value, then we say that : the signal x(t), has a dominant spectral component at frequency "f". This means that, a major portion of this signal is composed of frequency f. If the integration result is a small value, than this means that the signal does not have a major frequency component of f in it. If this integration result is zero, then the signal does not contain the frequency "f" at all.

It is of particular interest here to see how this integration works: The signal is multiplied with the sinusoidal term of frequency "f". If the signal has a high amplitude component of frequency "f", then that component and the sinusoidal term will coincide, and the product of them will give a (relatively) large value.

This shows that, the signal "x", has a major frequency component of "f".

However, if the signal does not have a frequency component of "f", the product will yield zero, which shows that, the signal does not have a frequency component of "f". If the frequency "f", is not a major component of the signal "x(t)", then the product will give a (relatively) small value. This shows that, the frequency component "f" in the signal "x", has a small amplitude, in other words, it is not a major component of "x". Now, note that the integration in the transformation equation (FT Eqn. 1) is over time. The left hand side of (1), however, is a function of frequency. Therefore, the integral in (1), is calculated for every value of f.

The information provided by the integral, corresponds to all time instances, since the integration is from minus infinity to plus infinity over time. It follows that no matter where in time the component with frequency "f" appears, it will affect the result of the integration equally as well. In other words, whether the frequency component "f" appears at time t1 or t2, it will have the same effect on the integration. This is why Fourier transform is not suitable if the signal has time varying frequency, i.e., the signal is non-stationary. If only the signal has the frequency component "f" at all times (for all "t" values), then the result obtained by the Fourier transform makes sense. Note that the Fourier transform tells whether a certain frequency component exists or not. This information is independent of where in time this component appears. It is therefore very important to know whether a signal is stationary or not, prior to processing it with the FT.

The Short Term Fourier Transforms: There is only a minor difference between STFT and FT. In STFT, the signal is divided into small enough segments, where these segments (portions) of the signal can be assumed to be stationary. For this purpose, a window function "w" is chosen. The width of this window must be equal to the segment of the signal where its stationary is valid.

This window function is first located to the very beginning of the signal. That is, the window function is located at t=0. Let's suppose that the width of the window is "T" s. At this time instant (t=0), the window function will overlap with the first T/2 seconds (I will assume that all time units are in seconds). The window function and the signal are then multiplied. By doing this, only the first T/2 seconds of the signal is being chosen, with the appropriate weighting of the window

(if the window is a rectangle, with amplitude "1", then the product will be equal to the signal). Then this product is assumed to be just another signal, whose FT is to be taken. In other words, FT of this product is taken, just as taking the FT of any signal.

The result of this transformation is the FT of the first T/2 seconds of the signal. If this portion of the signal is stationary, as it is assumed, then there will be no problem and the obtained result will be a true frequency representation of the first T/2 seconds of the signal. The next step would be shifting this window (for some t1 seconds) to a new location, multiplying with the signal, and taking the FT of the product. This procedure is followed; until the end of the signal is reached by shifting the window with "t1" seconds intervals.

The following definition of the STFT summarizes all the above explanations in one line:

$$STFT_x^{(a)}(t, f) = \int [x(t) \cdot w^*(t - \tau)] \cdot e^{-j2\pi f \tau} d\tau \dots (4)$$

Please look at the above equation carefully. x(t) is the signal itself, w(t) is the window function, and * is the complex conjugate. As you can see from the equation, the STFT of the signal is nothing but the FT of the signal multiplied by a window function. For every 't' and 'f' a new STFT coefficient is computed (Correction: The "t" in the parenthesis of STFT should be "t". I will correct this soon. I have just noticed that I have mistyped it).

The following figure may help you to understand this a little better:

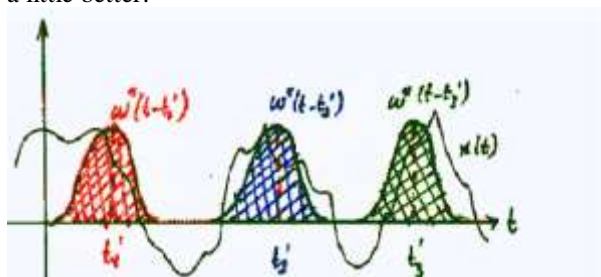


Figure 1

The Gaussian-like functions in color are the windowing functions. The red one shows the window located at t=t1, the blue shows t=t2, and the green one shows the window located at t=t3. These will correspond to three different FTs at three different times. Therefore, we will obtain a true time-frequency representation (TFR) of the signal.

Probably the best way of understanding this would be looking at an example. First of all, since our transform is a function of both 'time' and 'frequency' (unlike FT, which is a function of frequency only), the transform would be two dimensional (three, if you count the amplitude too). Let's take a non-stationary signal, such as the following one in figure 9:

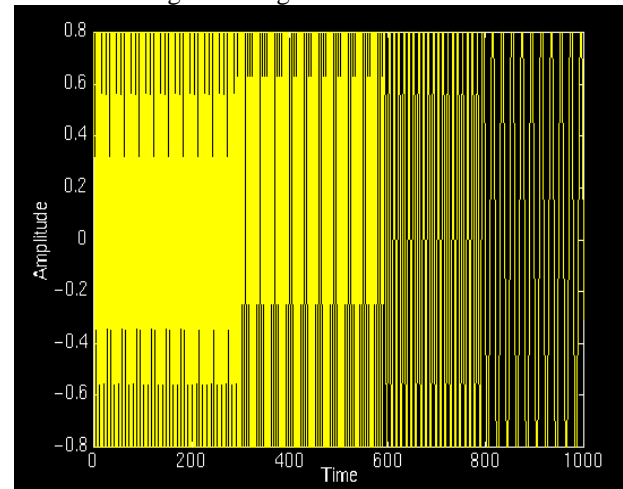


Figure 2

In this signal, there are four frequency components at different times. The interval 0 to 250 ms is a simple sinusoid of 300 Hz, and the other 250 ms intervals are sinusoids of 200 Hz, 100 Hz, and 50 Hz, respectively.

Apparently, this is a non-stationary signal. Now, let's look at its STFT:

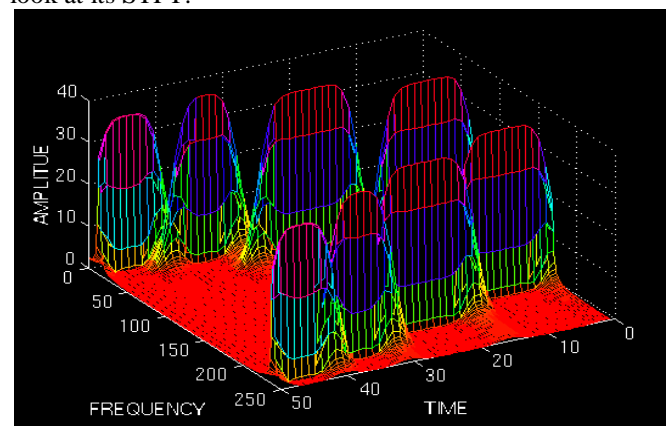


Figure 3

As expected, this is two dimensional plots (3 dimensional, if you count the amplitude too). The "x" and "y" axes are time and frequency, respectively. Please, ignore the numbers on the axes, since they are normalized in some respect, which is not of any

interest to us at this time. Just examine the shape of the time-frequency representation.

First of all, note that the graph is symmetric with respect to midline of the frequency axis. Remember that, although it was not shown, FT of a real signal is always symmetric, since STFT is nothing but a windowed version of the FT, it should come as no surprise that STFT is also symmetric in frequency. The symmetric part is said to be associated with negative frequencies, an odd concept which is difficult to comprehend, fortunately, it is not important; it suffices to know that STFT and FT are symmetric.

What is important, are the four peaks; note that there are four peaks corresponding to four different frequency components. Also note that, unlike FT, these four peaks are located at different time intervals along the time axis. Remember that the original signal had four spectral components located at different times. You may wonder, since STFT gives the TFR of the signal, why do we need the wavelet transform. The implicit problem of the STFT is not obvious in the above example. Of course, an example that would work nicely was chosen on purpose to demonstrate the concept.

The problem with STFT is the fact whose roots go back to what is known as the Heisenberg Uncertainty Principle. This principle originally applied to the momentum and location of moving particles, can be applied to time-frequency information of a signal. Simply, this principle states that one cannot know the exact time-frequency representation of a signal, i.e., one cannot know what spectral components exist at what instances of times. What one can know is the time intervals in which certain band of frequencies exist, which is a resolution problem. The problem with the STFT has something to do with the width of the window function that is used. To be technically correct, this width of the window function is known as the support of the window. If the window function is narrow than its known as compactly supported. This terminology is more often used in the wavelet world, as we will see later.

Recall that in the FT there is no resolution problem in the frequency domain, i.e., we know exactly what frequencies exist; similarly we there is no time resolution problem in the time domain, since we know the value of the signal at every instant of time. Conversely, the time resolution in the FT, and the frequency resolution in the time domain are zero, since we have no information about them. What gives the

perfect frequency resolution in the FT is the fact that the window used in the FT is its kernel, the $\exp\{j\omega t\}$ function, which lasts at all times from minus infinity to plus infinity. Now, in STFT, our window is of finite length, thus it covers only a portion of the signal, which causes the frequency resolution to get poorer. What I mean by getting poorer is that, we no longer know the exact frequency components that exist in the signal, but we only know a band of frequencies that exist:

In FT, the kernel function, allows us to obtain perfect frequency resolution, because the kernel itself is a window of infinite length. In STFT is window is of finite length, and we no longer have perfect frequency resolution. You may ask, why don't we make the length of the window in the STFT infinite, just like as it is in the FT, to get perfect frequency resolution? Well, than you loose all the time information, you basically end up with the FT instead of STFT. To make a long story real short, we are faced with the following dilemma:

If we use a window of infinite length, we get the FT, which gives perfect frequency resolution, but no time information. Furthermore, in order to obtain the stationary, we have to have a short enough window, in which the signal is stationary. The narrower we make the window, the better the time resolution, and better the assumption of stationary, but poorer the frequency resolution:

- Narrow window \implies good time resolution, poor frequency resolution.
- Wide window \implies good frequency resolution, poor time resolution.

Stationary wavelet transforms

The discrete stationary wavelet transform (SWT) is a un decimated version of DWT. The main idea is to average several detailed co-efficient which are obtained by decomposition of the input signal without downs sampling. This approach can be interpreted as a repeated application of the standard DWT method for different time shifts.

The Stationary wavelet transform (SWT) is similar to the dwt except the signal is never sub sampled and instead the filters are up sampled at each level of decomposition.

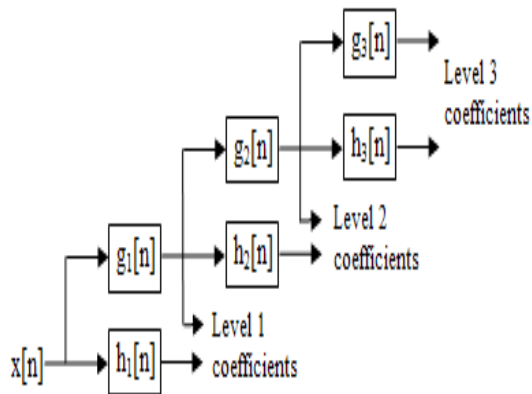


Figure 4: 3-level SWT filter bank

Each level's filters are up-sampled versions of the previous.



Figure 5

SWT filters The SWT is an inherently redundant scheme as each set of coefficients contains the same number of samples as the input – so for a decomposition of N levels there are a redundancy of 2N.

Wavelet reconstruction: The discrete wavelet transform can be used to analyze, or decompose, signals and images. This process is called decomposition or analysis. The other half of the story is how those components can be assembled back into the original signal without loss of information. This process is called reconstruction, or synthesis. The mathematical manipulation that effects synthesis is called the inverse discrete wavelet transforms (IDWT). To synthesize a signal using Wavelet Toolbox™ software, we reconstruct it from the wavelet coefficients:

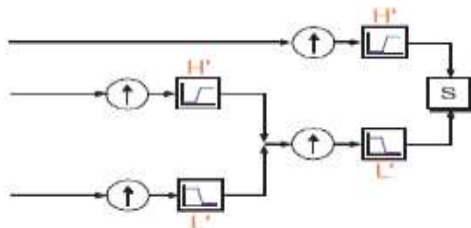


Figure 5

Where wavelet analysis involves filtering and down-sampling, the wavelet reconstruction process consists of up-sampling and filtering. Up-sampling is the process of lengthening a signal component by inserting zeros between samples:

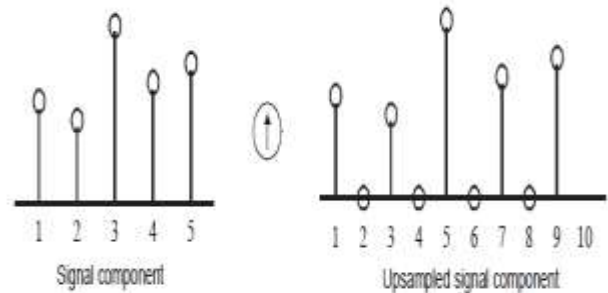


Figure 6

The toolbox includes commands, like 'idwt' and 'waverec', that perform single-level or multilevel reconstruction, respectively, on the components of one-dimensional signals. These commands have their two-dimensional analogs, idwt2 and waverec2.

Reconstruction filters: The filtering part of the reconstruction process also bears some discussion, because it is the choice of filters that is crucial in achieving perfect reconstruction of the original signal. The down-sampling of the signal components performed during the decomposition phase introduces a distortion called aliasing. It turns out that by carefully choosing filters for the decomposition and reconstruction phases that are closely related (but not identical), we can “cancel out” the effects of aliasing. The low- and high-pass decomposition filters (L and H), together with their associated reconstruction filters (L' and H'), form a system of what is called quadrature mirror filters:

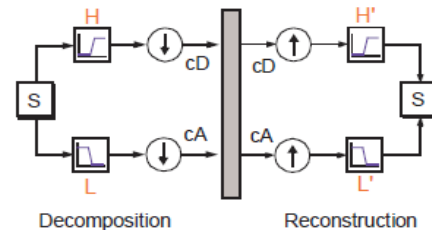


Figure 6

Reconstructing approximations and details: We have seen that it is possible to reconstruct our original signal from the coefficients of the approximations and details.

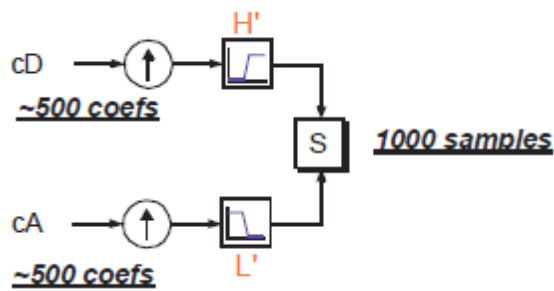


Figure 7

It is also possible to reconstruct the approximations and details themselves from their coefficient vectors. As an example, let's consider how we would reconstruct the first-level approximation A1 from the coefficient vector cA1. We pass the coefficient vector cA1 through the same process we used to reconstruct the original signal. However, instead of combining it with the level-one detail cD1, we feed in a vector of zeros in place of the detail coefficients vector:

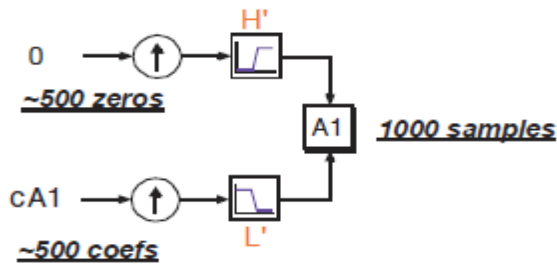


Figure 8

The process yields a reconstructed approximation A1, which has the same length as the original signal S and which is a real approximation of it. Similarly, we can reconstruct the first-level detail D1, using the analogous process:

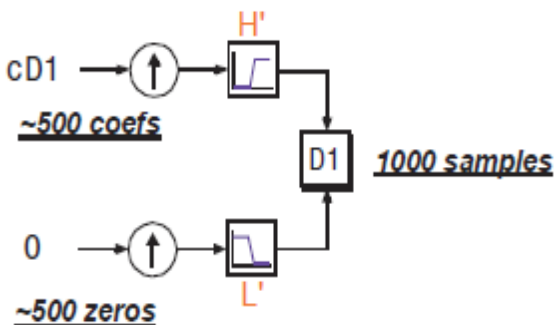


Figure 9

The reconstructed details and approximations are true constituents of the original signal. In fact, we find when we combine them that

$$A_1 + D_1 = S$$

Equation (6)

Note that the coefficient vectors cA1 and cD1 — because they were produced by down-sampling and are only half the length of the original signal — cannot directly be combined to reproduce the signal. It is necessary to reconstruct the approximations and details before combining them. Extending this technique to the components of a multilevel analysis, we find that similar relationships hold for all the reconstructed signal constituents. That is, there are several ways to reassemble the original signal:

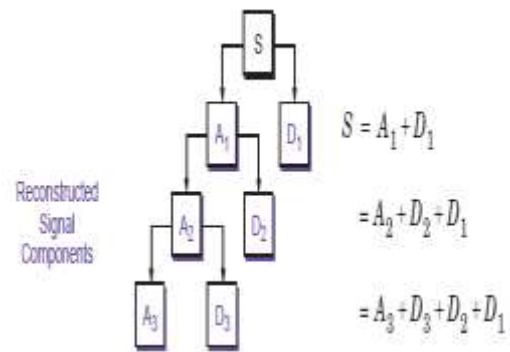


Figure 10

Wavelet families: Several families of wavelets that have proven to be especially useful. Some wavelet families are

- Haar
- Daubachies
- Biorthogonal
- Coiflets
- Symlets
- Morlet
- Mexicanhat
- Meyer
- Other real wavelets
- complex wavelets

Haar: Any discussion of wavelets begins with Haar wavelet, the first and simplest. Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelet as Daubechies db1.

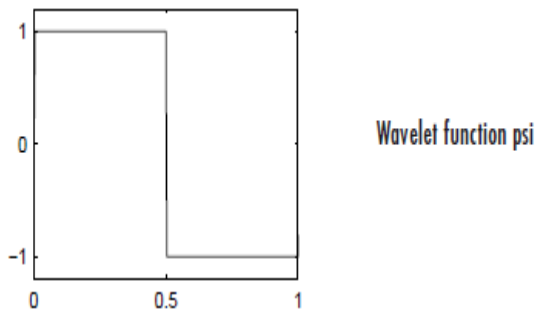


Figure 11

Daubechies: Ingrid Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets — thus making discrete wavelet analysis practicable. The names of the Daubechies family wavelets are written dbN, where N is the order, and db the “surname” of the wavelet. The db1 wavelet, as mentioned above, is the same as Haar wavelet. Here are the wavelet functions psi of the next nine members of the family:

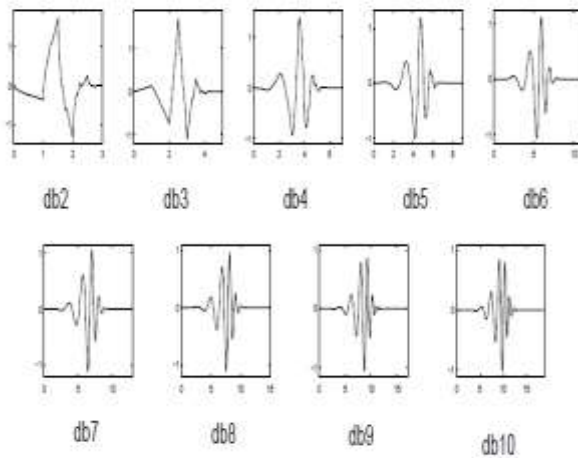


Figure 12

Biorthogonal: This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side) and the other for reconstruction (on the right side) instead of the same single one, interesting properties are derived.

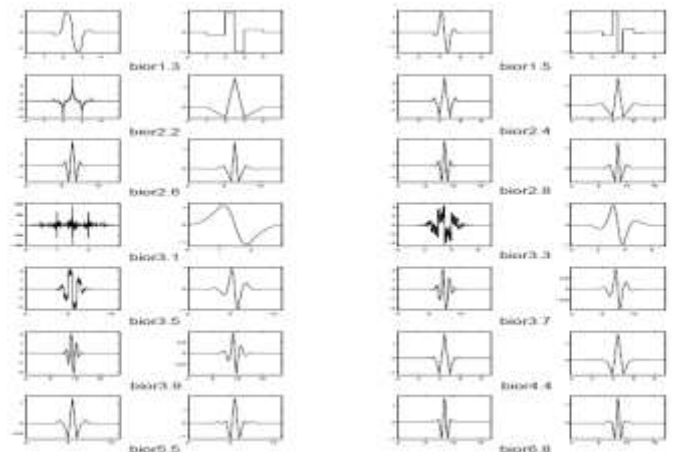


Figure 13

Coiflets: Built by I. Daubechies at the request of R. Coifman. The wavelet function has $2N$ moments equal to 0 and the scaling function has $2N-1$ moments equal to 0. The two functions have a support of length $6N-1$. You can obtain a survey of the main properties of this family by typing `waveinfo('coif')` from the MATLAB command line

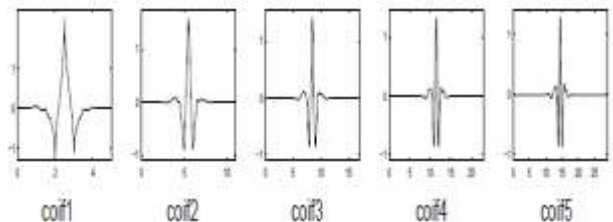


Figure 14

Symlets: The symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar. Here are the wavelet functions psi.

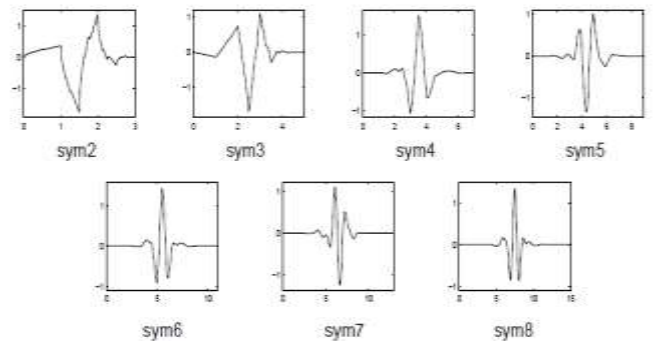


Figure 15

Morlet: This wavelet has no scaling function, but is explicit.

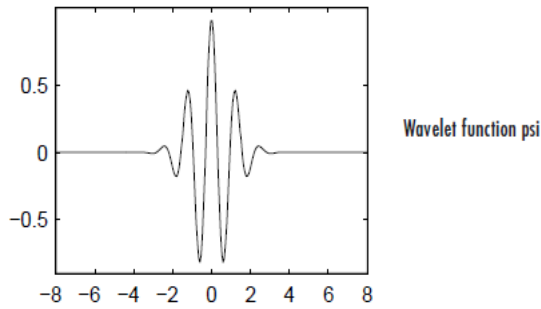


Figure 16

Mexican Hat: This wavelet has no scaling function and is derived from a function that is proportional to the second derivative function of the Gaussian probability density function.

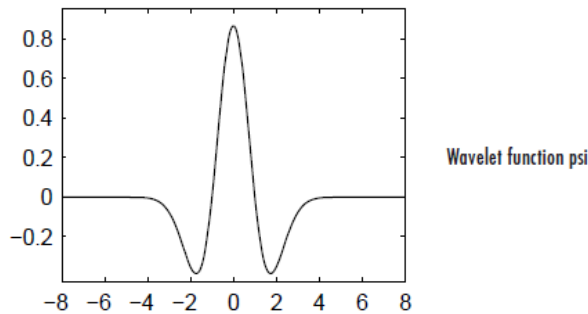


Figure 16

Meyer: The Meyer wavelet and scaling function are defined in the frequency domain.

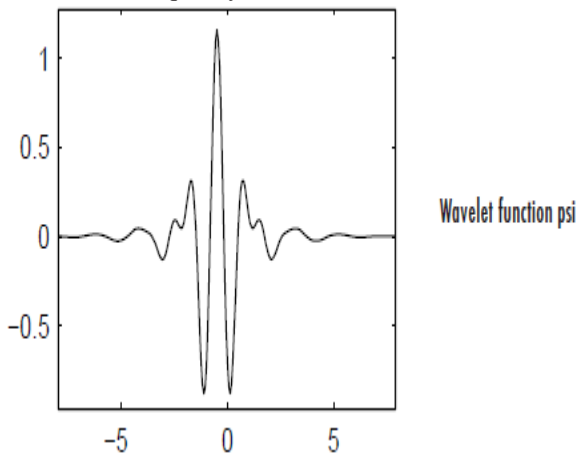


Figure 17: Meyer Wavelet

Other Real Wavelets: Some other real wavelets are available in the toolbox:

- ❖ Reverse Bi-orthogonal
- ❖ Gaussian derivatives family
- ❖ FIR based approximation of the Meyer wavelet

Complex Wavelets: Some complex wavelet families are available in the toolbox:

- ❖ Gaussian derivatives, Frequency B-Spline
- ❖ Morlet, Shannon

Simulation results

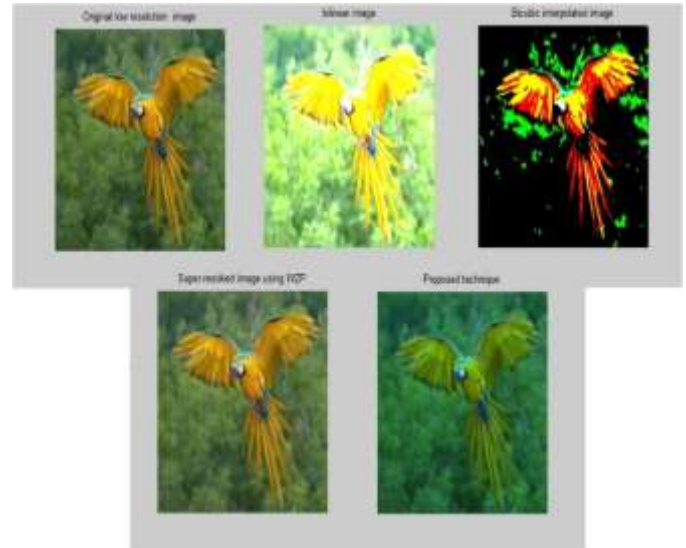


Figure 18: (a) Original low resolution Bird image.
 (b) Bilinear Image.
 (c) Bi-cubic interpolated image.
 (d) Super resolved image using WZP.
 (e) Proposed technique.

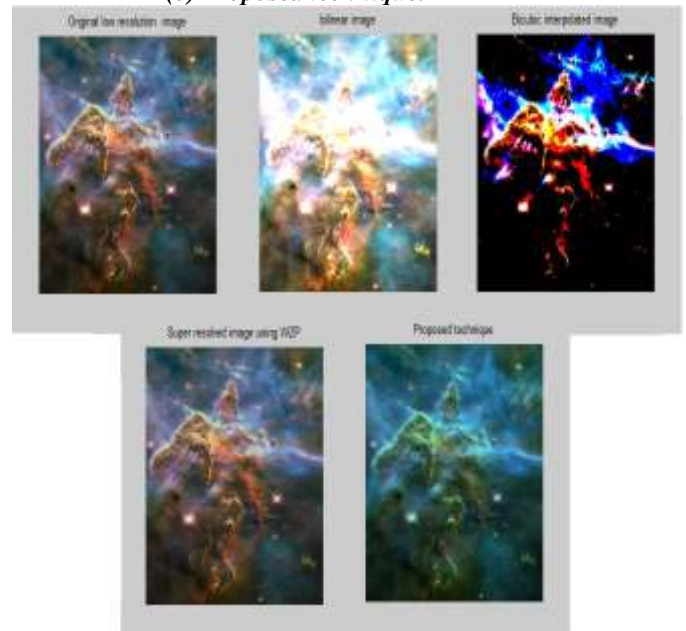


Figure 19: (a) Original low resolution Nebulae image.
 (b) Bilinear Image.
 (c) Bi-cubic interpolated image.

(d) Super resolved image using WZP.
 (e) Proposed technique.



Figure 20: (a) Original low resolution House image.

(b) Bilinear Image.
 (c) Bi-cubic interpolated image.
 (d) Super resolved image using WZP.
 (e) Proposed technique.

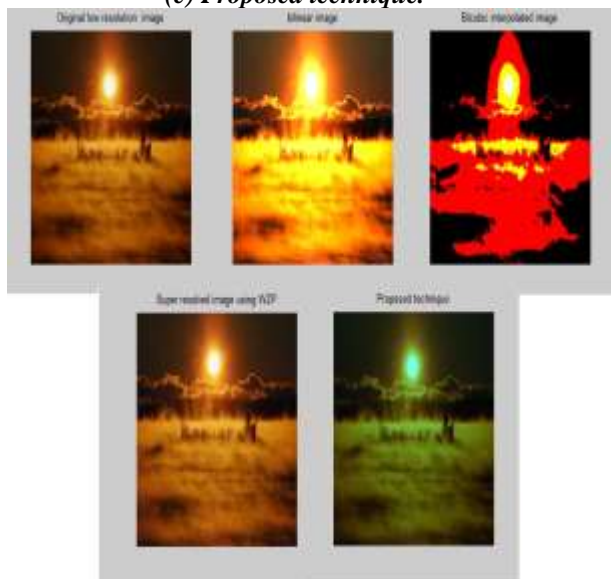


Figure 21: (a) Original low resolution Sunset image.

(b) Bilinear Image.
 (c) Bi-cubic interpolated image.
 (d) Super resolved image using WZP.
 (e) Proposed technique.

Bilinear	Bi-cubic	WZP	Proposed Method	File Name
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27.8195	29.8929	35.7398	35.8006	Sunset
29.9443	31.9591	37.5773	37.7966	Bird
30.7013	32.7232	38.0464	38.3842	Nebulae
27.2191	29.2454	34.3577	34.7264	House

Table 1: PSNR values' comparison table for RGB images

Conclusion

This work proposes an image resolution enhancement technique based on the interpolation of the high frequency sub bands obtained by DWT, correcting the high frequency sub-band estimation by using SWT high frequency sub bands, and the input image. The proposed technique uses DWT to decompose an image into different sub bands, and then the high frequency sub band images have been interpolated. The interpolated high frequency sub band coefficients have been corrected by using the high frequency sub bands achieved by SWT of the input image. An original image is interpolated with half of the interpolation factor used for interpolation the high frequency sub bands. Afterwards all these images have been combined using IDWT to generate a super resolved imaged. The proposed technique has been tested on well-known benchmark images, where their PSNR and visual results show the superiority of proposed technique over the conventional and state-of-art image resolution enhancement techniques.

References

1. L. Yi-bo, X. Hong, and Z. Sen-yue, "The wrinkle generation method for facial reconstruction based on extraction of partition wrinkle line features and fractal interpolation," in Proc. 4th Int. Conf. Image Graph., Aug. 22–24, 2007, pp. 933–937.
2. Y. Rener, J. Wei, and C. Ken, "Downsample-based multiple description coding and post-processing of decoding", in Proc. 27th Chinese Control Conf., Jul. 16–18, 2008, pp. 253–256.
3. H. Demirel, G. Anbarjafari, and S. Izadpanahi, "Improved motionbased localized super resolution technique using discrete wavelet transform for low resolution video enhancement," in Proc. 17th Eur. Signal Process. Conf., Glasgow, Scotland, Aug. 2009, pp. 1097–1101.
4. Y. Piao, I. Shin, and H. W. Park, "Image resolution enhancement using inter-subband correlation in wavelet domain," in Proc. Int.

- Conf. Image Process., 2007, vol. 1, pp. I-445–448.
5. H. Demirel and G. Anbarjafari, “Satellite image resolution enhancement using complex wavelet transform,” *IEEE Geoscience and Remote Sensing Letter*, vol. 7, no. 1, pp. 123–126, Jan. 2010.
 6. C. B. Atkins, C. A. Bouman, and J. P. Allebach, “Optimal image scaling using pixel classification,” in *Proc. Int. Conf. Image Process.*, Oct. 7–10, 2001, vol. 3, pp. 864–867.
 7. W. K. Carey, D. B. Chuang, and S. S. Hemami, “Regularity-preserving image interpolation,” *IEEE Trans. Image Process.*, vol. 8, no. 9, pp. 1295–1297, Sep. 1999.
 8. S. Mallat, *A Wavelet Tour of Signal Processing*, 2nd ed. New York: Academic, 1999.
 9. J. E. Fowler, “The redundant discrete wavelet transform and additive noise,” Mississippi State ERC, Mississippi State University, Tech. Rep. MSSU-COE-ERC-04-04, Mar. 2004.
 10. X. Li and M. T. Orchard, “New edge-directed interpolation,” *IEEE Trans. Image Process.*, vol. 10, no. 10, pp. 1521–1527, Oct. 2001.
 11. K. Kinebuchi, D. D. Muresan, and R. G. Baraniuk, “Waveletbased statistical signal processing using hidden Markov models,” in *Proc. Int. Conf. Acoust., Speech, Signal Process.*, 2001, vol. 3, pp. 7–11.
 12. S. Zhao, H. Han, and S. Peng, “Wavelet domain HMT-based image super resolution,” in *Proc. IEEE Int. Conf. Image Process.*, Sep. 2003, vol. 2, pp. 933–936.
 13. A. Temizel and T. Vlachos, “Wavelet domain image resolution enhancement using cycle-spinning,” *Electron. Lett.*, vol. 41, no. 3, pp. 119–121, Feb. 3, 2005.
 14. A. Temizel and T. Vlachos, “Image resolution upscaling in the wavelet domain using directional cycle spinning,” *J. Electron. Imag.*, vol. 14, no. 4, 2005.
 15. G. Anbarjafari and H. Demirel, “Image super resolution based on interpolation of wavelet domain high frequency subbands and the spatial domain input image,” *ETRI J.*, vol. 32, no. 3, pp. 390–394, Jun. 2010.
 16. A. Temizel, “Image resolution enhancement using wavelet domain hidden Markov tree and coefficient sign estimation,” in *Proc. Int. Conf. Image Process.*, 2007, vol. 5, pp. V-381–384